GENERALIZED SPECTRAL DOMAIN METHOD FOR MULTI-CONDUCTOR PRINTED LINES AND ITS APPLICATION TO TUNABLE SUSPENDED MICROSTRIPS*

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ABSTRACT

An efficient method is developed for obtaining propagation characteristics of microstrip-line type structures in which a number of conductors are located on various interfaces. Specific computations have been carried out for suspended microstrip line structures with tuning conductive septums. A number of data useful for design are included.

Introduction

The spectral domain technique developed by Itoh and Mittra has been applied to a number of microstrip-line structures.[1,2] It is an efficient numerical technique having several advantages over many other methods. However, to date, this technique has been applied only to the structures in which center conductors (strips) are located on one of the dielectric interfaces, e.g. the air-substrate interface.

This paper reports a modification of the spectral domain technique which can handle the structures in which a number of conductors are placed on various interfaces (Fig. 1). The original version is not capable of solving such structures. Also, formulation in this paper is quite general and requires no structural symmetry to exist. Before discussing the technique, we will describe the motivation of the present work.

Recently, several attempts have been made to increase design flexibility of MIC structures by introducing additional conductors on interfaces different from the one on which the original strips are located. Aikawa reported the use of grounded septums located on the lower side of the substrate in the coupled suspended line[3] (Fig. 2). He has successfully developed tight couplers by adjusting the width of septums without which such couplers were extremely difficult to realize. Such composite structures are difficult to analyze and the design procedure based on slowly converging numerical methods is prohibitively expensive because there are more structural parameters to be adjusted than in conventional structures. Hence, development of an efficient analysis method is needed.

The principal purpose of the paper is to present a formulation for general structures (Fig. 1). Numerical results are presented for the suspended microstrip with two grounded septums. Most of the data are for single strip case (S = 0 in Fig. 2) as extensive data for coupled lines will be reported elsewhere.

Formulation

In this paper, we restrict ourselves to cases where the quasi-TEM approximation is valid, although the present method can readily be extended to a more rigorous dispersion analysis. Under this assumption, we only need to solve Poisson's equation in the cross section subject to appropriate boundary conditions. Instead of solving such a problem directly in the xy coordinate, we introduce the Fourier transform of the potential

$$\tilde{\phi}(n, y) = \int_0^L \phi(x, y) \sin \frac{n\pi}{2L} x \, dx \tag{1}$$

When the Poisson's equation is Fourier transformed, its solution in the i-th layer is

$$\tilde{\phi}_i(n, y) = A_i(n) \sinh \frac{n\pi}{2L} y + B_i(n) \cosh \frac{n\pi}{2L} y \tag{2}$$

The interface conditions are also Fourier transformed, and the transformed conditions are used to eliminate all the A_i's and B_i's. After mathematical derivation for this process is completed, one obtains the following coupled algebraic equations.

$$\sum_{j=1}^{N} \tilde{G}_{ij}(n) \tilde{\rho}_j(n) = \tilde{\phi}_{Vi} + \tilde{\phi}_{oi}, \quad i = 1, 2, \ldots, N \tag{3}$$

where $\tilde{G}_{ij}$'s are known, $\tilde{\rho}_j$'s are the transforms of unknown charge distributions at the j-th interface. $\tilde{\phi}_{Vi}$'s are transforms of the given potential distributions on the strips at the i-th interface whereas $\tilde{\phi}_{oi}$'s are transforms of unknown potential distributions outside the strips at the i-th interface. When there is no strip at the j-th surface, the j-th equation vanishes and the j-th term on the left-hand side becomes zero as $\tilde{\rho}_j$ is zero for such j.

Notice that (3) is an N × N matrix equation in contrast to a set of N × N coupled integral equations appearing in convential space domain formulations which contain convolution integrals. $\tilde{G}_{ij}$ is actually the transform of the Green's function G_{ij} which determines the potential at the i-th interface due to the unit charge at the j-th interface. Also (3) contains a total of 2N unknowns, $\tilde{\rho}_j$ and $\tilde{\phi}_{oi}$. However, N unknowns, $\tilde{\phi}_{oi}$, can be eliminated in the solution process and one can solve (3) only for N unknown $\tilde{\rho}_j$'s. To this end we apply Galerkin's method to (3). First we expand $\tilde{\rho}_j$ in terms of known basis functions

$$\tilde{\rho}_j(n) = \sum_{s=1}^{S_j} \sum_{p=1}^{P_j} c_p^s \tilde{\rho}_{jp}^s(n) \tag{4}$$

where S_j is the number of strips at the j-th interface.

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$\tilde{\rho}_{jp}$ is the transform of an assumed charge distribution on the s-th strip at the j-th interface.

Substituting (4) into (3) and taking the inner products of the resulting equations with $\tilde{\rho}_{iq}^{v}$, q = 1, ..., P_i and v = 1, ..., S_i, one obtains the following $\sum_{j=1}^{N} P_j S_j \times \sum_{j=1}^{N} P_j S_j$ matrix equations for c_p^s

$$\sum_{j=1}^{N} \sum_{s=1}^{S_j} \sum_{p=1}^{P_j} K_{qp}^{vs}(i, j)\, c_p^s = Y_q^v(i) \qquad \begin{array}{l} i = 1, 2, \ldots, N \\ v = 1, \ldots, S_i \\ q = 1, \ldots, P_i \end{array} \tag{5}$$

where K_{qp}^{vs} and Y_q^v are known quantities and can be computed quite efficiently once $\tilde{\rho}_{jp}^{s}$ is selected. Once c_p^s's are obtained by solving (5), the charge distribution on the s-th strip at the j-th interface can readily be computed from

$$\rho_j(x) = \sum_{s=1}^{S_j} \sum_{p=1}^{P_j} c_p^s\, \rho_{jp}^s(x) \tag{6}$$

where ρ_{jp}^s is the assumed charge distribution from which $\tilde{\rho}_{jp}^{s}$ was analytically derived.

Although (5) may seem complicated, in most cases it results in small size matrix, because for a reasonably accurate answer P_j only needs to be unity or at most two. For instance, when only one strip each is located at two different interfaces (S_j = 1, N = 2), the size of the matrix is either 2 × 2 or 4 × 4.

Results for the Suspended Microstrips with Septums

Numerical results were obtained for both single and coupled suspended microstrip with septums (Fig. 2). First the accuracy of the method was checked by comparing our results with those reported by Aikawa[3] who used a finite difference technique. As shown in Fig. 3, the agreement is quite satisfactory.

A number of data are presented here for a single suspended line with symmetric septums. Figs. 4 and 5 present characteristic impedance and normalized guide wavelength, respectively, versus the width of the strip for a number of septum widths. It is seen from Fig. 5 that for large a, the guide wavelength λ_g becomes smaller as the strip width is increased. On the other hand, when a is reduced λ_g takes a maximum at some W. The reason for this phenomenon may be as follows: When a is large, the effect of the air portion (Region 1) to the field distribution is reduced. As W is increased, most of the flux lies in the dielectric region, causing λ_g to be small. For small a, λ_g resembles that of the conventional suspended line. As W is increased, the effect of air becomes more important and λ_g increases until the coupling between the strip and the septums becomes dominant. After such a situation a larger amount of flux moves into the dielectric region and λ_g becomes smaller again.

Fig. 6 shows characteristic impedance versus the septum width a for three different dielectric materials. The strip width W is fixed. It is clear that Z can be adjusted over a wide range by varying a. This feature is quite attractive in MIC application because in suspended line the fabrication of low impedance lines is often difficult.[4]

Conclusions

We presented a general method, based on spectral domain approach, for multi-conductor printed lines for MIC. Numerical examples are given for the suspended microstrip with grounded septums. This structure is considered useful for MIC application, because propagation characteristics can be adjusted by septums which add one more degree of freedom in the design.

The numerical method presented here is applicable to a wide range of problems and has several advantageous features. (1) The method is numerically efficient. (2) No convolution integrals are involved. (3) The size of matrix is quite small.

References

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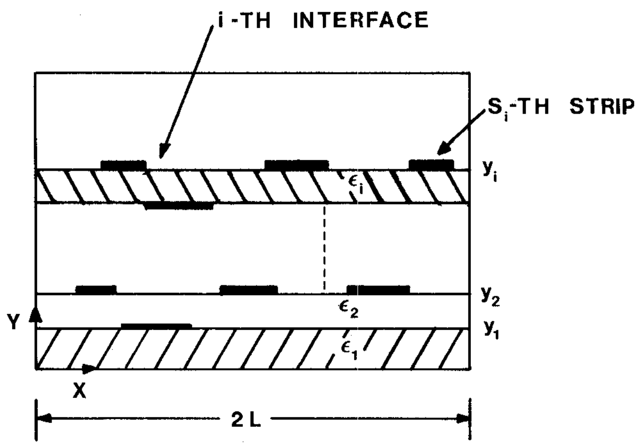

Fig. 1. Cross section of shielded multi-conductor printed line.

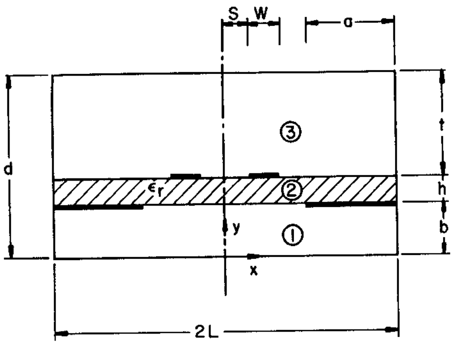

Fig. 2. Suspended microstrip line with septums.

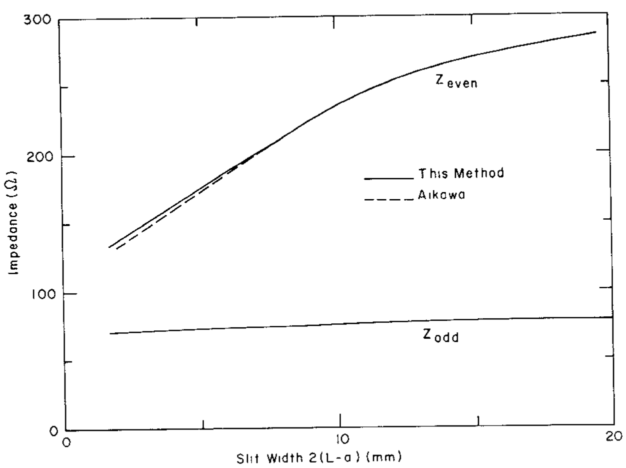

Fig. 3. Comparison of results with those by Aikawa. $\varepsilon_r = 2.4$, S = 0.335mm, W = 1.48mm, L = 16.4 mm, t = 16.4mm, h = 1.64mm, b = 8.2mm.

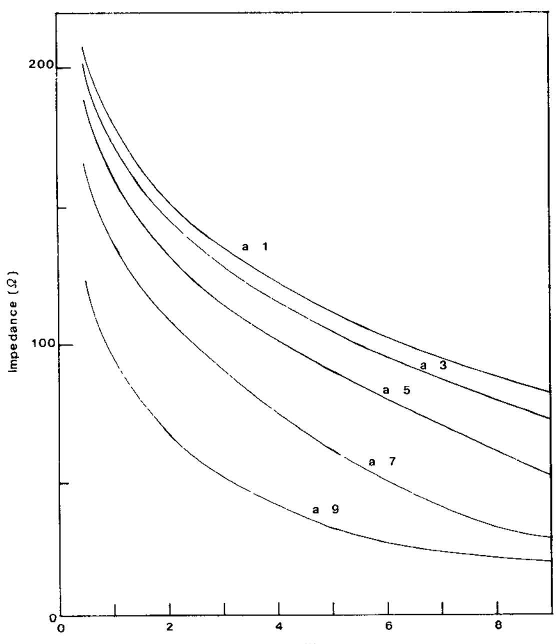

Fig. 4. Characteristic impedance vs. the strip width. $\varepsilon_r = 3.8$, S/h = 0, L/h = b/h = t/h = 10

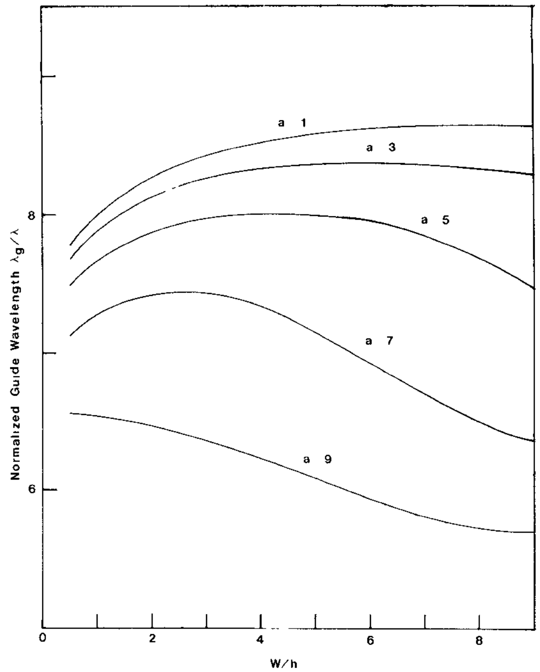

Fig. 5. Normalized guide wavelength vs. the strip width. $\varepsilon_r = 3.8$, S/h = 0, L/h = b/h = t/h = 10

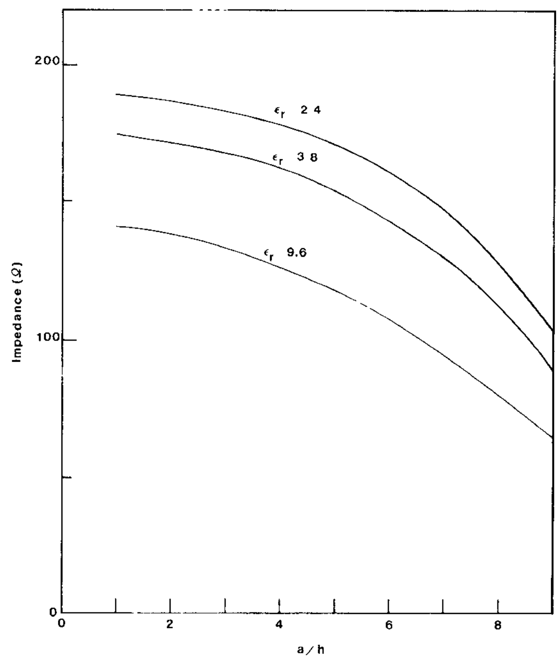

Fig. 6. Characteristic impedance vs. the septum width. S/h = 0, W/h = 1.2, L/h = b/h = t/h = 10